Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-03-18

Conceptual questions

Problems marked with "**" are more difficult and/or more tangential to the main course content.

Question 1. Suppose that f is a function such that $\partial f/\partial x > 0$ always and $\partial f/\partial y < 0$ always. One can use a Riemann sum to approximate $\int_0^1 \int_0^1 f(x,y)\,\mathrm{d}x\,\mathrm{d}y$ by breaking up the square into (for example) nine equally sized smaller squares.

(a) If the goal is to guarantee an *overestimate* of the integral, where should the sample points be chosen?

(b) What if we want to *underestimate*?

Question 2. **Let f and g be two single-variable functions. Suppose we approximate $\int_0^6 f(x)\,\mathrm{d}x$ by breaking up the interval into three equally sized pieces, and taking $1, 2, 6$ as sample points. Suppose we approximate $\int_0^6 g(x)\,\mathrm{d}x$ by breaking up the interval into two equally sized pieces, taking $2, 4$ as sample points.

If we multiply these approximations, we get an approximation for

$$\int_0^6 f(x)\,\mathrm{d}x \int_0^6 g(x)\,\mathrm{d}x = \int_0^6 \int_0^6 f(x)g(y)\,\mathrm{d}x\,\mathrm{d}y.$$

Explain how this approximation is also a Riemann sum. (What's the corresponding decomposition of the integration region? What are the sample points?)

Computations

Problem 1. Reverse the order of integration:

$$\int_0^1 \int_0^{2y} f(x,y)\,\mathrm{d}x\,\mathrm{d}y + \int_1^3 \int_0^{3-y} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_?^? \int_?^? f(x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

Problem 2. Evaluate the following double integral:

$$\int_0^1 \int_0^1 e^{\max(x^2,y^2)}\,\mathrm{d}x\,\mathrm{d}y.$$

You should break up the integral into two pieces: one for the region where $x \geq y$ and the other for the region where $x \leq y$.